1. OPERATOR DWUCZĄSTKOWY $\hat{f}^{(l,l')}$ dla l, l'-tej cząstki

np. $\hat{f}^{(l,l')} = \hat{f}_e(\hat{\vec{r}}_l, \hat{\vec{r}}_{l'}) \to$ oddziaływanie elektrostatyczne

$= \hat{f}_m(\hat{\vec{S}}_l, \hat{\vec{S}}_{l'}) \to$ — // — magnetyczne (spinowe)

Elementy macierzowe $\hat{f}^{(l,l')} \in \mathcal{H}_l \otimes \mathcal{H}_{l'}$ w bazie $\mathcal{H}_l \otimes \mathcal{H}_{l'}$ to

$$\hat{f}^{(l,l')} = \left(\hat{\mathbb{1}}^{(l)} \otimes \hat{\mathbb{1}}^{(l')}\right) \hat{f}^{(l,l')} \left(\hat{\mathbb{1}}^{(l)} \otimes \hat{\mathbb{1}}^{(l')}\right) =$$

$$= \left(\sum_{j_1=1}^{k} |\alpha_{j_1}^{(l)}\rangle\langle\alpha_{j_1}^{(l)}| \otimes \sum_{j_2=1}^{k} |\alpha_{j_2}^{(l')}\rangle\langle\alpha_{j_2}^{(l')}| \right) \hat{f}^{(l,l')} \left(\sum_{j_1=1}^{k} |\alpha_{j_1}^{(l)}\rangle\langle\alpha_{j_1}^{(l)}| \otimes \sum_{j_2=1}^{k} |\alpha_{j_2}^{(l')}\rangle\langle\alpha_{j_2}^{(l')}| \right) =$$

$$= \sum_{j_1,j_2=1}^{k} |\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')})^{rr} (\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')}| \hat{f}^{(l,l')} \sum_{j_3,j_4=1}^{k} |\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')})^{rr} (\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')}| =$$

$$= \sum_{j_1,j_2,j_3,j_4=1}^{k} |\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')})^{rr} (\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')}| \underbrace{{}^{r}(\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')}| \hat{f}^{(l,l')} |\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')})^{r}}_{= f_{j_1 j_2 j_3 j_4}} \qquad (1)$$

$\to$ NIE ZALEŻY od l ani l' !

a) układ N cząstek

$$\hat{F} = \frac{1}{2} \sum_{\substack{l,l'=1 \\ l \neq l'}}^{N} \hat{f}^{(l,l')} = \sum_{\substack{l,l'=1 \\ l < l'}}^{N} \hat{f}^{(l,l')} \qquad (2)$$

bo 2 razy suma przebiega po tej samej parze cząstek

Z (1) i (2) mamy, że

$$\hat{F} = \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{j_1,j_2,j_3,j_4=1}^{k} \left(\hat{\mathbb{1}}^{(1)} \otimes \ldots \otimes |\alpha_{j_1}^{(l)}\rangle\langle\alpha_{j_3}^{(l)}| \otimes \ldots \otimes |\alpha_{j_2}^{(l')}\rangle\langle\alpha_{j_4}^{(l')}| \otimes \ldots \otimes \hat{\mathbb{1}}^{(N)}\right) f_{j_1 j_2 j_3 j_4}$$

$$= \sum_{j_1,j_2,j_3,j_4=1}^{k} \hat{F}_{j_1 j_2 j_3 j_4} \qquad (3)$$

$\to$ policzmy $\hat{F}_{j_1 j_2 j_3 j_4} |n_1, \ldots, n_k\rangle$

$$\hat{F}_{j_1 j_2 j_3 j_4} |n_1, \ldots, n_k\rangle = \hat{F}_{j_1 j_2 j_3 j_4} |\alpha_{j'_1}, \ldots, \alpha_{j'_N}\rangle_+^{nr} \overset{(3),\ \text{normalizacja}}{=}$$

$$= f_{j_1 j_2 j_3 j_4} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \left(\hat{\mathbb{1}}^{(1)} \otimes \ldots \otimes |\alpha_{j_1}^{(l)}\rangle\langle\alpha_{j_3}^{(l)}| \otimes \ldots \otimes |\alpha_{j_2}^{(l')}\rangle\langle\alpha_{j_4}^{(l')}| \otimes \ldots \otimes \hat{\mathbb{1}}^{(N)}\right) \frac{|\alpha_{j'_1}, \ldots, \alpha_{j'_N})_+^{nr}}{\sqrt{n_1! \cdot \ldots \cdot n_k!}} =$$

kombinacja cz. rozróżnialnych

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{n_1! \cdot \ldots \cdot n_k!}} \frac{1}{\sqrt{N!}} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} |\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')})^{rr} (\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')}| \sum_{P \in \Pi(N)} |\alpha_{j'_{P(1)}}^{(1)}, \ldots \alpha_{j'_{P(l)}}^{(l)}, \ldots, \alpha_{j'_{P(l')}}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} =$$

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{P \in \Pi(N)} |\alpha_{j_1}^{(l)}, \alpha_{j_2}^{(l')})^{rr} (\alpha_{j_3}^{(l)}, \alpha_{j_4}^{(l')} | \alpha_{j'_{P(1)}}^{(1)}, \ldots \alpha_{j'_{P(l)}}^{(l)}, \ldots, \alpha_{j'_{P(l')}}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} =$$

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{P \in \Pi(N)} \delta_{j_3 j'_{P(l)}} \delta_{j_4 j'_{P(l')}} |\alpha_{j'_{P(1)}}^{(1)}, \ldots, \alpha_{j_1}^{(l)}, \ldots, \alpha_{j_2}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} = \ldots$$

- W tym nowym stanie: $n_{j_1} \leftarrow n_{j_1}+1$ oraz $n_{j_3} \leftarrow n_{j_3}-1$ i $n_{j_2} \leftarrow n_{j_2}+1$ oraz $n_{j_4} \leftarrow n_{j_4}-1$
- $\sum_{P \in \Pi(N)} \ldots = \sum_{m=1}^{N} \sum_{m'=1}^{N} \sum_{\substack{P \in \Pi(N) \\ P(l)=m \\ P(l')=m'}} \ldots$
- u nas $l \neq l'$, więc siłą rzeczy $m \neq m'$

$$\ldots = \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{\substack{m,m'=1 \\ m \neq m'}}^{N} \sum_{\substack{P \in \Pi(N) \\ P(l)=m \\ P(l')=m'}} \delta_{j_3 j'_m} \delta_{j_4 j'_{m'}} |\alpha_{j'_{P(1)}}^{(1)}, \ldots, \alpha_{j_1}^{(l)}, \ldots, \alpha_{j_2}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} =$$

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \underbrace{\sum_{\substack{m,m'=1 \\ m \neq m'}}^{N} \delta_{j_3 j'_m} \delta_{j_4 j'_{m'}}}_{} \sum_{\substack{P \in \Pi(N) \\ P(l)=m \\ P(l')=m'}} |\alpha_{j'_{P(1)}}^{(1)}, \ldots, \alpha_{j_1}^{(l)}, \ldots, \alpha_{j_2}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} = \ldots$$

Mamy sumę niezależną od permutacji P.

- Sprawdzamy, ile razy w starym stanie cząstka będzie w stanie $|\alpha_{j_3}\rangle \rightarrow$ odp. n_{j_3}
- A ile razy cząstki będą w stanie $|\alpha_{j_4}\rangle$?
  - $\rightarrow 1^\circ$ $j_4 \neq j_3 \rightarrow$ mamy różne stany, więc mimo warunku $m \neq m'$ uda się zliczyć wszystkie cząstki $\rightarrow$ odp. n_{j_4}
  - $\rightarrow 2^\circ$ $j_4 = j_3 \rightarrow$ liczymy tak samo jak dla $|\alpha_{j_3}\rangle$, ale warunek $m \neq m'$ obniża wynik o 1 $\rightarrow$ odp. $n_{j_4}-1$.

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} n_{j_3}(n_{j_4} - \delta_{j_3 j_4}) \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{\substack{P \in \Pi(N) \\ P(l)=m \\ P(l')=m'}} |\alpha_{j'_{P(1)}}^{(1)}, \ldots, \alpha_{j_1}^{(l)}, \ldots, \alpha_{j_2}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} =$$

$\left.\begin{matrix} P \in \Pi(N) \\ P(l)=m \\ P(l')=m' \end{matrix}\right\}$ Takich permutacji jest $(N-2)!$

$$= \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{N! n_1! \ldots n_k!}} n_{j_3}(n_{j_4} - \delta_{j_3 j_4}) \sum_{\substack{l,l'=1 \\ l<l'}}^{N} \sum_{\substack{P \in \Pi(N) \\ P(l)=m \\ P(l')=m'}} |\alpha_{j'_{P(1)}}^{(1)}, \ldots, \alpha_{j_1}^{(l)}, \ldots, \alpha_{j_2}^{(l')}, \ldots, \alpha_{j'_{P(N)}}^{(N)})^{r} = \ldots$$

- $\sum_{\substack{l,l'=1 \\ l<l'}}^{N}$ przebiega po każdej możliwej parze liczb z $\{1,..,N\}$ dokładnie 1 raz $\to$ tych par jest $N(N-1)/2$

$\sum_{\substack{P\in\pi(N) \\ P(l)=m \\ P(l')=m'}}$ $\to$ mamy $(N-2)!$ permutacji $\longrightarrow$ $N!/2$ wyrazów zatem

notacja Focka

$$\ldots = \frac{f_{j_1j_2j_3j_4}}{\sqrt{N!n_1!\ldots n_k!}}\frac{n_{j_3}(n_{j_4}-\delta_{j_3j_4})}{2}\sum_{Q\in\pi(N)}|\alpha^{(1)}_{j''_{Q(1)}},\ldots,\alpha^{(N)}_{j''_{Q(N)}})^{r} =$$

nowy stan

$$= \frac{f_{j_1j_2j_3j_4}}{\sqrt{n_1!\ldots n_k!}}\frac{n_{j_3}(n_{j_4}-\delta_{j_3j_4})}{2}\sqrt{n_1!\ldots(n_{j_1}+1)!\ldots(n_{j_2}+1)!\ldots(n_{j_3}-1)!\ldots(n_{j_4}-1)!\ldots n_k!}$$

$$\cdot|n_1,\ldots,n_{j_1}+1,\ldots,n_{j_2}+1,\ldots,n_{j_3}-1,\ldots,n_{j_4}-1,\ldots,n_k\rangle \qquad (*)$$

W $(*)$ milcząco zakładamy, że $j_1\neq j_2\neq j_3\neq j_4$. Ale w ogólności

1° $j_1=j_2=j_3=j_4=j$

$$\to \hat{F}_{jjjj}|n_1,\ldots,n_k\rangle \overset{(*)}{=} \frac{f_{jjjj}}{\cancel{\sqrt{n_1!\ldots n_k!}}}\frac{n_j(n_j-1)}{2}\cancel{\sqrt{n_1!\ldots n_k!}}|n_1,\ldots,n_k\rangle =$$

$$= \frac{1}{2}f_{jjjj}(\hat{a}^+_j\hat{a}_j\hat{a}^+_j\hat{a}_j - \hat{a}^+_j\hat{a}_j)|n_1,\ldots,n_k\rangle =$$

$$= \frac{1}{2}f_{jjjj}(\hat{a}^+_j(\hat{a}^+_j\hat{a}_j+\cancel{1})\hat{a}_j - \cancel{\hat{a}^+_j\hat{a}_j})|n_1,\ldots,n_k\rangle =$$

$$= \frac{1}{2}f_{jjjj}\hat{a}^+_j\hat{a}^+_j\hat{a}_j\hat{a}_j|n_1,\ldots,n_k\rangle$$

2° $j=j_1=j_2=j_3\neq j_4=j'$

$$\to \hat{F}_{jjjj'}|n_1,\ldots,n_k\rangle \overset{(*)}{=} \frac{f_{jjjj'}}{\sqrt{n_1!\ldots n_k!}}\frac{n_jn_{j'}}{2}\sqrt{n_1!\ldots(n_j+1)!\ldots(n_{j'}-1)!\ldots n_k!}$$

$$\cdot|n_1,\ldots,n_j+1,\ldots,n_{j'}-1,\ldots,n_k\rangle =$$

$$= \frac{f_{jjjj'}}{2\cancel{\sqrt{n_{j'}}}}n_j\sqrt{n_{j'}}\sqrt{n_j+1}|n_1,\ldots,n_j+1,\ldots,n_{j'}-1,\ldots,n_k\rangle =$$

$$= \frac{1}{2}f_{jjjj'}n_j\sqrt{n_{j'}}\sqrt{n_j+1}|n_1,\ldots,n_j+1,\ldots,n_{j'}-1,\ldots,n_k\rangle =$$

reguły komutacji, $j\neq j'$

$$= \frac{1}{2}f_{jjjj'}n_j\hat{a}^+_j\hat{a}_{j'}|n_1,\ldots,n_k\rangle = \frac{1}{2}f_{jjjj'}\hat{a}^+_j\hat{a}_{j'}\hat{a}^+_j\hat{a}_j|n_1,\ldots,n_k\rangle =$$

liczba

$$= \frac{1}{2}f_{jjjj'}\hat{a}^+_j\hat{a}^+_j\hat{a}_j\hat{a}_{j'}|n_1,\ldots,n_k\rangle$$

3° $j=j_1=j_2\neq j_3=j_4=j'$

$$\to \hat F_{jjj'j'}|n_1,\dots,n_k\rangle \overset{(*)}{=} \frac{f_{jjj'j'}}{\sqrt{n_1!\dots n_k!}}\,\frac{n_{j'}^2-n_{j'}}{2}\sqrt{n_1!\dots(n_j+2)!\dots(n_{j'}-2)!\dots n_k!}\cdot|n_1,\dots,n_j+2,\dots,n_{j'}-2,\dots,n_k\rangle=$$

$$=\frac{f_{jjj'j'}}{2}\sqrt{n_{j'}(n_{j'}-1)}\,\frac{\sqrt{(n_j+2)(n_j+1)}}{\cancel{\sqrt{n_{j'}(n_{j'}-1)}}}|n_1,\dots,n_j+2,\dots,n_{j'}-2,\dots,n_k\rangle=$$

$$=\frac{f_{jjj'j'}}{2}\,\hat a_j^+\hat a_j^+\sqrt{n_{j'}(n_{j'}-1)}\,|n_1,\dots,n_j,\dots,n_{j'}-2,\dots,n_k\rangle=$$

$$=\frac12 f_{jjj'j'}\,\hat a_j^+\hat a_j^+\hat a_{j'}\hat a_{j'}|n_1,\dots,n_k\rangle$$

4° $j=j_1\neq j_2=j_3=j_4=j'$

$$\to \hat F_{jj'j'j'}|n_1,\dots,n_k\rangle \overset{(*)}{=} \frac{f_{jj'j'j'}}{\sqrt{n_1!\dots n_k!}}\,\frac{n_{j'}(n_{j'}-1)}{2}\sqrt{n_1!\dots(n_j+1)!\dots(n_{j'}-1)!\dots n_k!}\cdot|n_1,\dots,n_j+1,\dots,n_{j'}-1,\dots,n_k\rangle=$$

$$=\frac{f_{jj'j'j'}}{2\cancel{\sqrt{n_{j'}}}}\sqrt{n_{j'}}(n_{j'}-1)\sqrt{n_j+1}\,|n_1,\dots,n_j+1,\dots,n_{j'}-1,\dots,n_k\rangle=$$

$$=\frac12 f_{jj'j'j'}\sqrt{n_{j'}}(n_{j'}-1)\,\hat a_j^+|n_1,\dots,n_j,\dots,n_{j'}-1,\dots,n_k\rangle=$$

$$=\frac12 f_{jj'j'j'}\,\hat a_j^+\hat a_{j'}(n_{j'}-1)|n_1,\dots,n_k\rangle=$$

$$=\frac12 f_{jj'j'j'}\left(\hat a_j^+\hat a_{j'}\hat a_{j'}^+\hat a_{j'}-\hat a_j^+\hat a_{j'}\right)|n_1,\dots,n_k\rangle=$$ reguły komutacji

$$=\frac12 f_{jj'j'j'}\left(\hat a_j^+(\cancel{1}+\hat a_{j'}^+\hat a_{j'})\hat a_{j'}-\cancel{\hat a_j^+\hat a_{j'}}\right)|n_1,\dots,n_k\rangle=$$

$$=\frac12 f_{jj'j'j'}\,\hat a_j^+\hat a_{j'}^+\hat a_{j'}\hat a_{j'}|n_1,\dots,n_k\rangle$$

5° $j=j_1=j_2\neq j_3\neq j_4$

$$\to \hat F_{jjj_3j_4}|n_1,\dots,n_k\rangle = \frac{f_{jjj_3j_4}}{\sqrt{n_1!\dots n_k!}}\,\frac{n_{j_3}n_{j_4}}{2}\times\sqrt{n_1!\dots(n_j+2)!\dots(n_{j_3}-1)!\dots(n_{j_4}-1)!}\cdot|n_1,\dots,n_j+2,\dots,n_{j_3}-1,\dots,n_{j_4}-1,\dots,n_k\rangle=$$

$$=\frac{f_{jjj_3j_4}}{2}\,\frac{\sqrt{(n_j+2)(n_j+1)}}{\cancel{\sqrt{n_{j_3}n_{j_4}}}}\sqrt{n_{j_3}n_{j_4}}\,|n_1,\dots,n_j+2,\dots,n_{j_3}-1,\dots,n_{j_4}-1,\dots,n_k\rangle=$$

$$=\frac12 f_{jjj_3j_4}\,\hat a_j^+\hat a_j^+\hat a_{j_3}\hat a_{j_4}|n_1,\dots,n_k\rangle$$

6° $j_1\neq j_2=j_3=j\neq j_4$

$$\to \hat F_{j_1jjj_4}|n_1,\dots,n_k\rangle = \frac{f_{j_1jjj_4}}{\sqrt{n_1!\dots n_k!}}\,\frac{n_j n_{j_4}}{2}\sqrt{n_1!\dots(n_{j_1}+1)!\dots(n_{j_4}-1)!\dots n_k!}\cdot|n_1,\dots,n_{j_1}+1,\dots,n_{j_4}-1,\dots,n_k\rangle=$$

$$= \frac{f_{j_1 j j j_4}}{2} \frac{n_j \sqrt{n_{j_4}} \sqrt{n_{j_1}+1}}{\cancel{\sqrt{n_{j_4}}}} |n_1, \dots, n_{j_1}+1, \dots, n_{j_4}-1, \dots, n_k\rangle =$$

$$= \frac{1}{2} f_{j_1 j j j_4} n_j \hat{a}^+_{j_1} \hat{a}_{j_4} |n_1, \dots, n_k\rangle =$$ reguły komutacji

$$= \frac{1}{2} f_{j_1 j j j_4} \hat{a}^+_{j_1} \hat{a}^+_j \hat{a}_j \hat{a}_{j_4} |n_1, \dots, n_k\rangle$$

7° $j_1 \neq j_2 \neq j_3 = j_4 = j$

$$\to \hat{F}_{j_1 j_2 j j} |n_1, \dots, n_k\rangle = \frac{f_{j_1 j_2 j j}}{\sqrt{n_1! \dots n_k!}} \frac{n_j(n_j-1)}{2}$$

$$\times \sqrt{n_1! \dots (n_{j_1}+1)! \dots (n_{j_2}+1)! \dots (n_j-2)!}$$

$$\cdot |n_1, \dots, n_{j_1}+1, \dots, n_{j_2}+1, \dots, n_j-2, \dots, n_k\rangle =$$

$$= \frac{f_{j_1 j_2 j j}}{2} \frac{\sqrt{(n_{j_1}+1)(n_{j_2}+1)}}{\cancel{\sqrt{n_j(n_j-1)}}} \sqrt{n_j(n_j-1)}$$

$$\cdot |n_1, \dots, n_{j_1}+1, \dots, n_{j_2}+1, \dots, n_j-2, \dots, n_k\rangle =$$ reguły komutacji

$$= \frac{1}{2} f_{j_1 j_2 j j} \hat{a}^+_{j_1} \hat{a}^+_{j_2} \hat{a}_j \hat{a}_j |n_1, \dots, n_k\rangle$$

8° $j_1 \neq j_2 \neq j_3 \neq j_4$

$$\to \hat{F}_{j_1 j_2 j_3 j_4} = \frac{f_{j_1 j_2 j_3 j_4}}{\sqrt{n_1! \dots n_k!}} \frac{n_{j_3} n_{j_4}}{2}$$

$$\times \sqrt{n_1! \dots (n_{j_1}+1)! \dots (n_{j_2}+1)! \dots (n_{j_3}-1)! \dots (n_{j_4}-1)! \dots n_k!}$$

$$\cdot |n_1, \dots, n_{j_1}+1, \dots, n_{j_2}+1, \dots, n_{j_3}-1, \dots, n_{j_4}-1, \dots, n_k\rangle =$$

$$= \frac{f_{j_1 j_2 j_3 j_4}}{2} \frac{\sqrt{(n_{j_1}+1)(n_{j_2}+1)}}{\cancel{\sqrt{n_{j_3} n_{j_4}}}} \sqrt{n_{j_3} n_{j_4}}$$

$$\cdot |n_1, \dots, n_{j_1}+1, \dots, n_{j_2}+1, \dots, n_{j_3}-1, \dots, n_{j_4}-1, \dots, n_k\rangle =$$

$$= \frac{1}{2} f_{j_1 j_2 j_3 j_4} \hat{a}_{j_1} \hat{a}_{j_2} \hat{a}_{j_3} \hat{a}_{j_4} |n_1, \dots, n_k\rangle$$

Podsumowując, dla każdego $j_1, j_2, j_3, j_4 \in \{1, \dots, k\}$

$$\boxed{\hat{F}_{j_1 j_2 j_3 j_4} = \frac{1}{2} f_{j_1 j_2 j_3 j_4} \hat{a}^+_{j_1} \hat{a}^+_{j_2} \hat{a}_{j_3} \hat{a}_{j_4}} \qquad (4)$$

2. RÓWNANIA MAXWELLA

(1) $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ → prawo Gaussa dla pola elektrycznego

(2) $\nabla \cdot \vec{B} = 0$ → prawo Gaussa dla pola magnetycznego

(3) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ → prawo indukcji Faradaya

(4) $\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$ → prawo Ampere'a - Maxwella

Dla próżni $\vec{J} = \vec{0}$ i $\rho = 0$